Fully Generalized Reactivity (1) Synthesis

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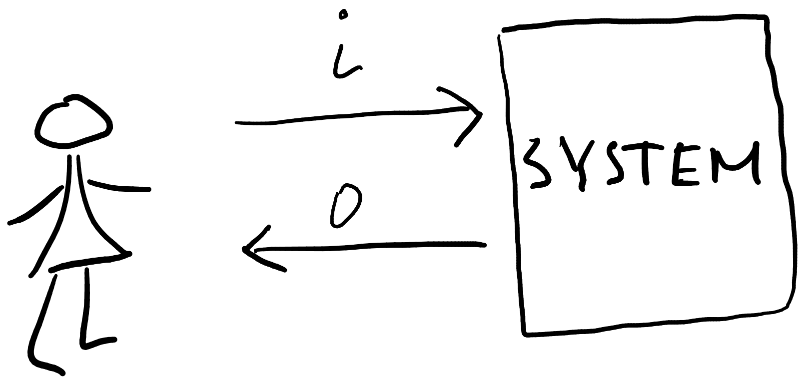
https://arxiv.org/abs/2402.02979

Church's Synthesis Problem



$$(i_0 o_0)(i_1 o_1)\ldots \in (I \times O)^\omega$$

Synthesis problem:
given: specification $\subseteq (I \times O)^\omega$
return: system whose every interaction $\in$ spec, else UNREALIZABLE

Synthesis Timeline

1962: Church's synthesis problem

1969: solved by RBL

1977: LTL introduced by Pnueli

1988: Safra's construction

1989: LTL synthesis is 2EXPTime-complete (PR)

2004: GR(1) synthesis (PPS) *impressive scalability*

⋮

Other approaches: safraless, bounded, anti-chain, safrafull, strix ...

Generalized Reactivity (1) Synthesis

GR(1)-safety $\wedge$ GR(1)-liveness

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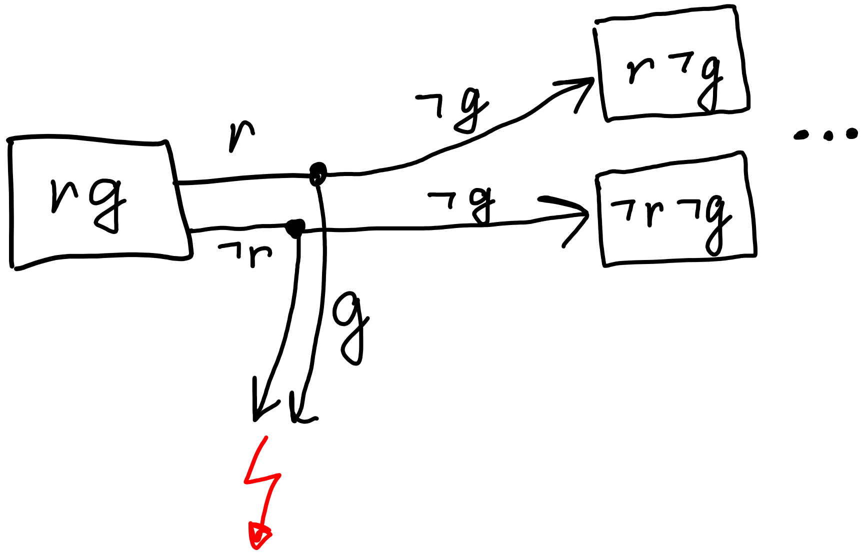
$G(r \wedge g \rightarrow X\neg g)$

(symbolic) arena



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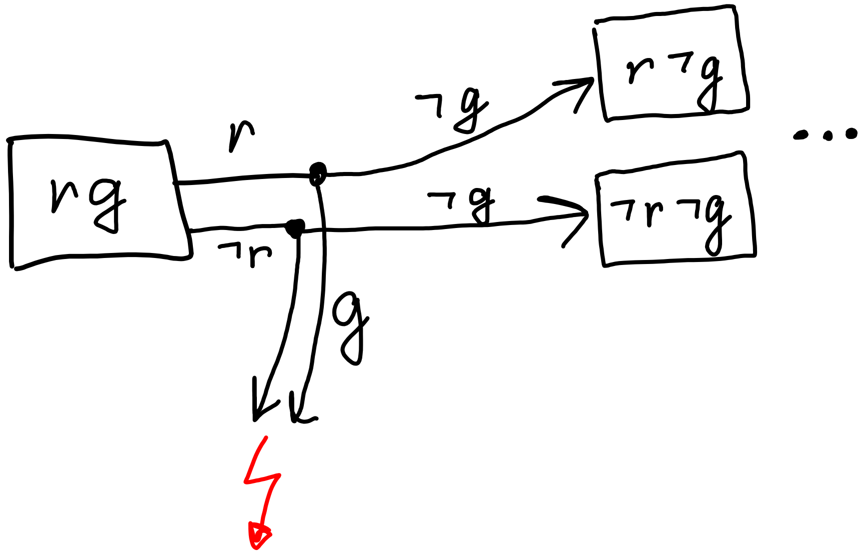

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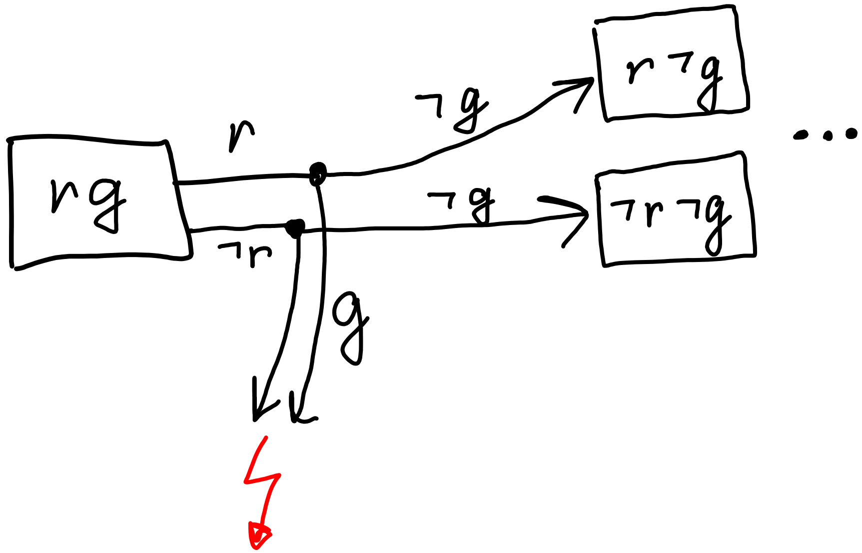

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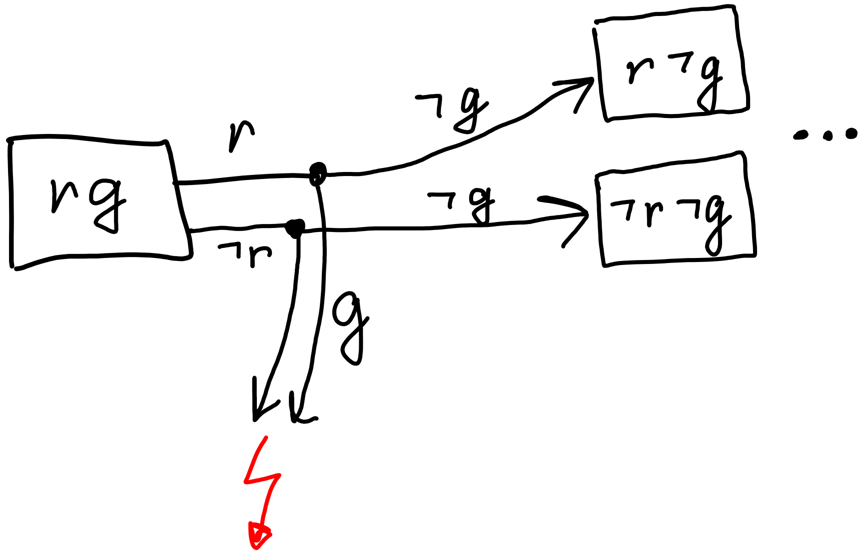

symbolic game with GR(1) objective $\bigwedge_i \mathsf{GF}\ldots \rightarrow \bigwedge_i \mathsf{GF}\ldots$ LTL

Our Problem: Symbolic Games with LTL Objectives

Given a symbolic game with LTL objective. Who wins the game?

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The symbolic representation should support:

- operations of union and conjunction on sets of label-vertex pairs
- enforceable predecessor $\Box\Diamond$: given a subset Φ of $2^{AP} \times V$ it returns

$$\Box\Diamond(\Phi) = \{v \in V \mid \forall i. \exists o : (i \cup o, \delta(v, i, o)) \in \Phi\}$$

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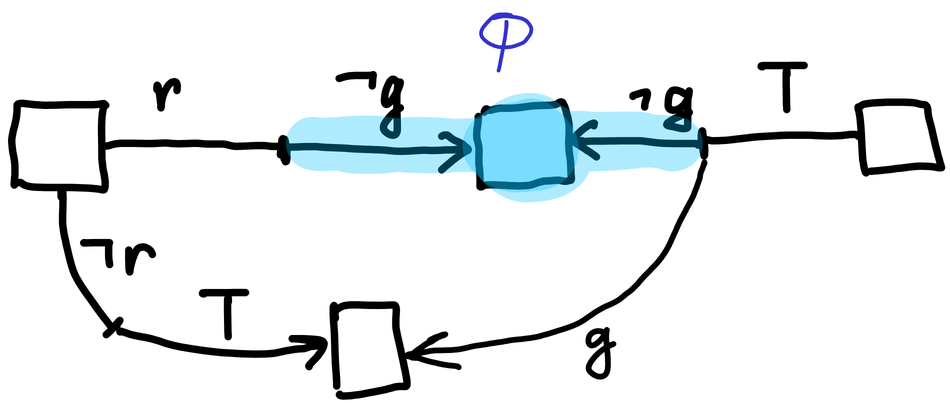

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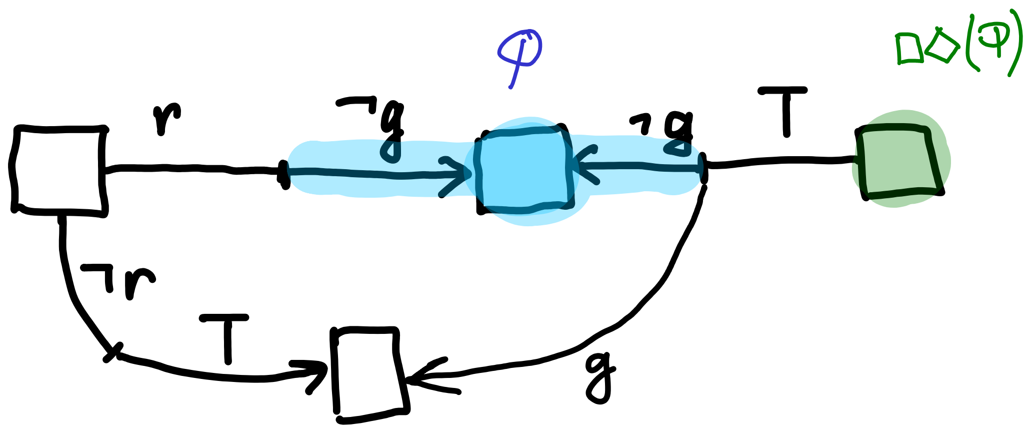

Our Approach to Solving Symbolic Games for LTL

- utilizes the canonical language representation COCOA of [ES]

our hidden motivation!

Our Approach to Solving Symbolic Games for LTL

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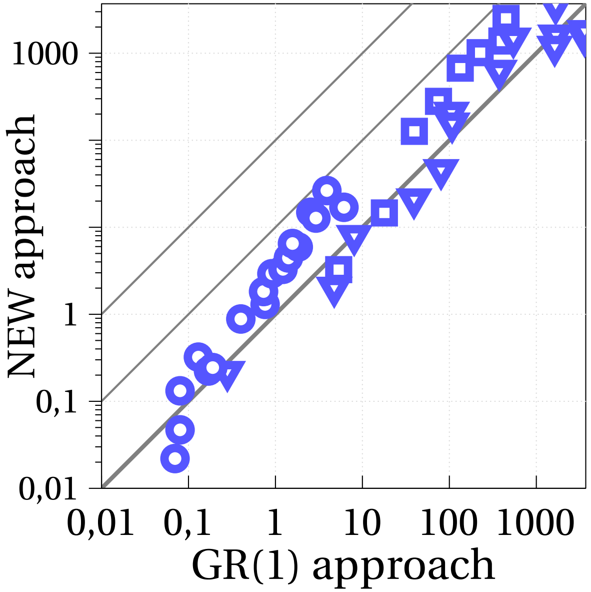
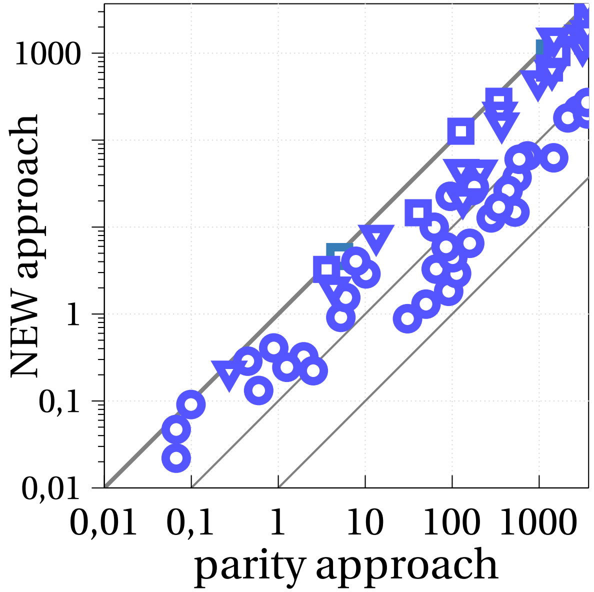
- is as fast as GR(1) approach:



- outperforms folklore approach:



GR(1) Synthesis Approach

$$\Phi_{\mathsf{GR(1)}}$$
$$\downarrow$$
fixpoint formula
$$\downarrow$$
evaluate it on the game graph

Folklore Approach to Symbolic LTL Games

$$\Phi_{\mathsf{LTL}}$$

$$\downarrow$$

parity automaton

$$\downarrow$$

fixpoint formula

$$\downarrow$$

evaluate it on the game graph

Our Approach to Symbolic LTL Games

Φ_{LTL}

$\downarrow$

parity automaton

$\downarrow$

COCOA

$\downarrow$ insight 1

alternating GFG
parity automaton

$\downarrow$ insight 2

fixpoint formula

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Canonical Chain of Co-Büchi Automata (COCOA)

A *chain of co-Büchi representation* of an ω-regular language L is a chain $L_1 \supset ... \supset L_n$ of co-Büchi languages such that a word w belongs to L if and only if the highest index i s.t. $w \in L_i$ is even or no such i exists.

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Σ^ω : $L_1 = \varnothing$

$\varnothing$: $(L_1 = \Sigma^\omega) \supset (L_2 = \varnothing)$

GFa: $(L_1 = L(\mathrm{FG}\overline{a})) \supset (L_2 = \varnothing)$

FGa: $(L_1 = \Sigma^\omega) \supset (L_2 = L(\mathrm{FG}a)) \supset (L_3 = \varnothing)$

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[ES] defined a canonical separation into such a chain.
[AK] defined a canonical form of GFG co-Büchi automata.

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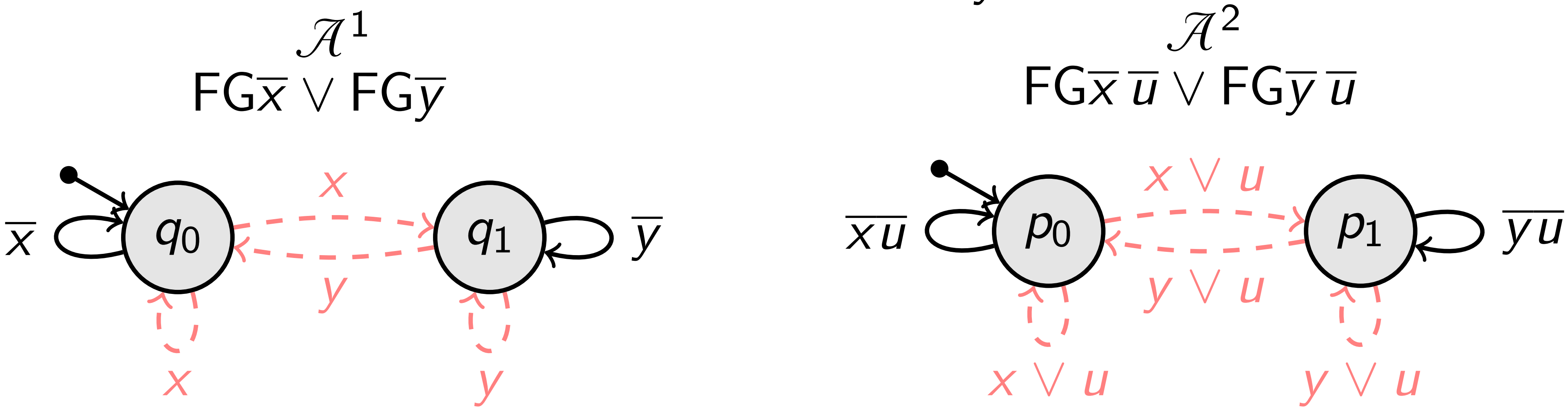
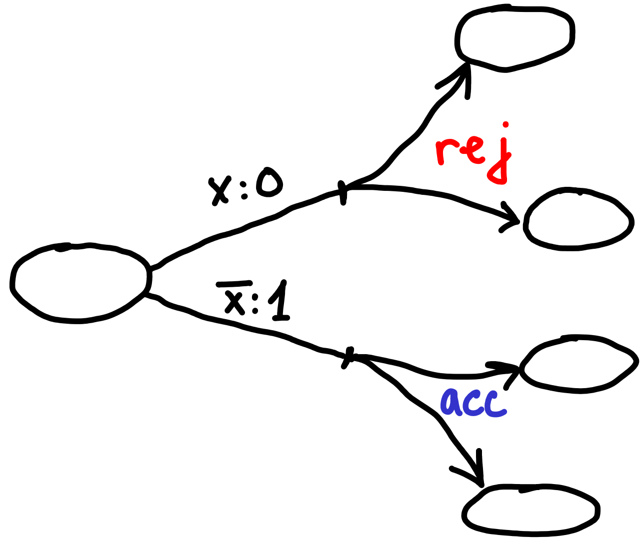
$\mathsf{GF}u \rightarrow \mathsf{GF}x \wedge \mathsf{GF}y$



COCOA → Alternating GFG Automaton

$$\mathcal{A} = \left(\Sigma,\ Q,\ q_0,\ \delta : Q \times \Sigma \to 2^{Q} \times \mathbb{N} \times \{rej, acc\}\right)$$



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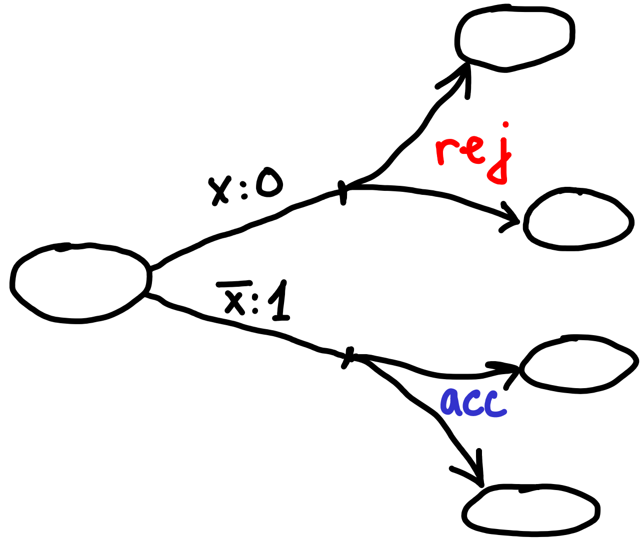
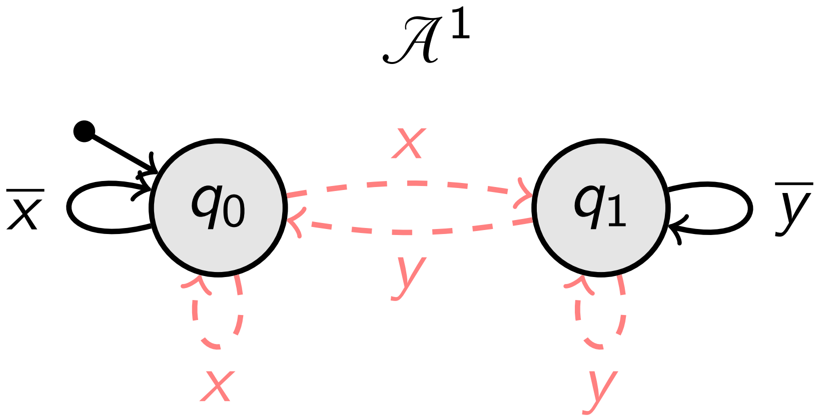
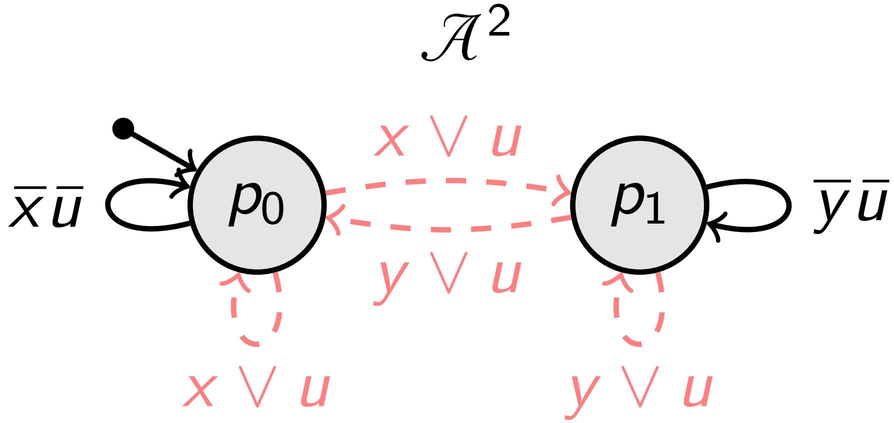
Alternating automaton:
a word w is *accepted* by $\mathcal{A}$ iff
the acceptor has a strategy$_w$
for resolving its nondeterminism
to produce a winning play.



GFG:
the acceptor has a uniform strategy,
which does not know the whole word
(only the current prefix).

COCOA → Alternating GFG Automaton

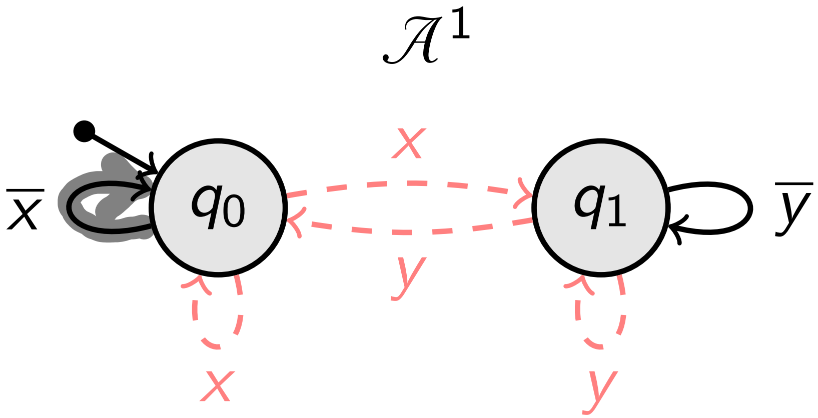


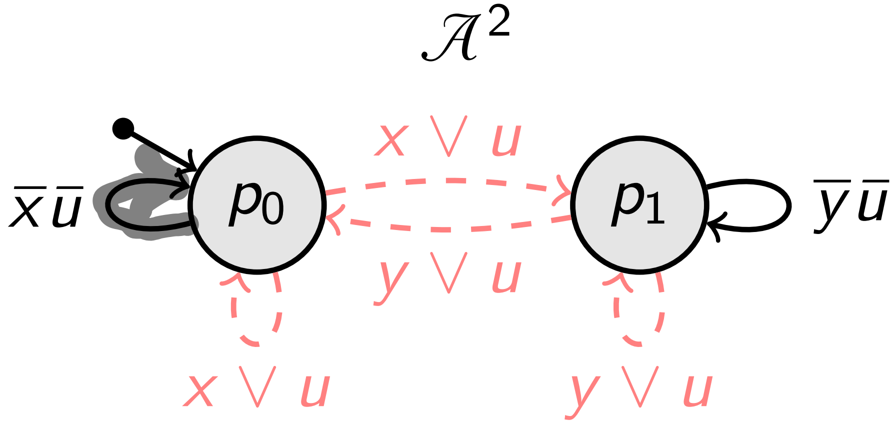


$(\mathcal{A}^3)$
$\emptyset$

A-GFG:

COCOA $\rightarrow$ Alternating GFG Automaton





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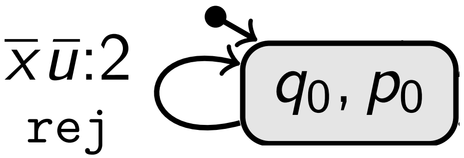

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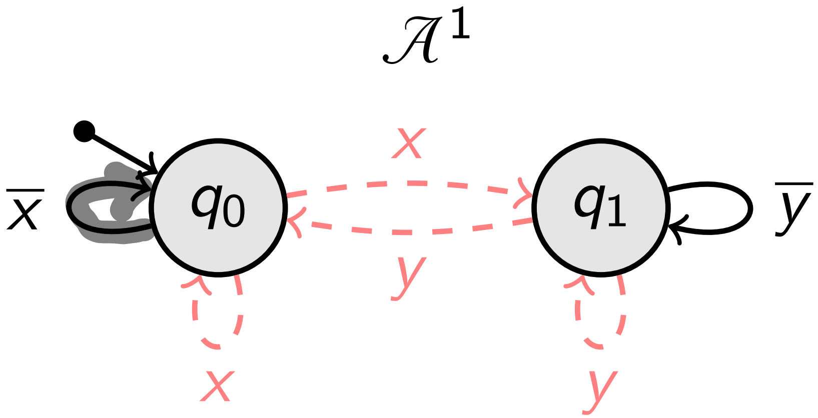

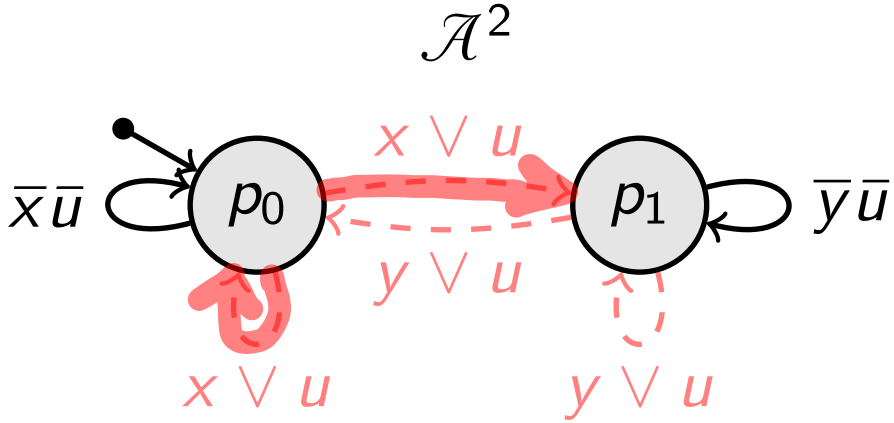

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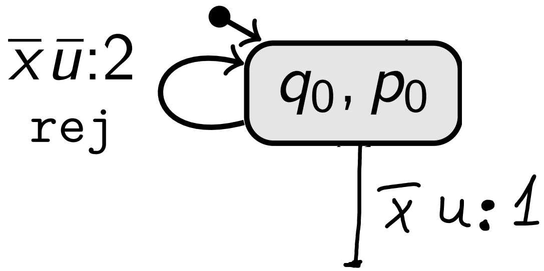

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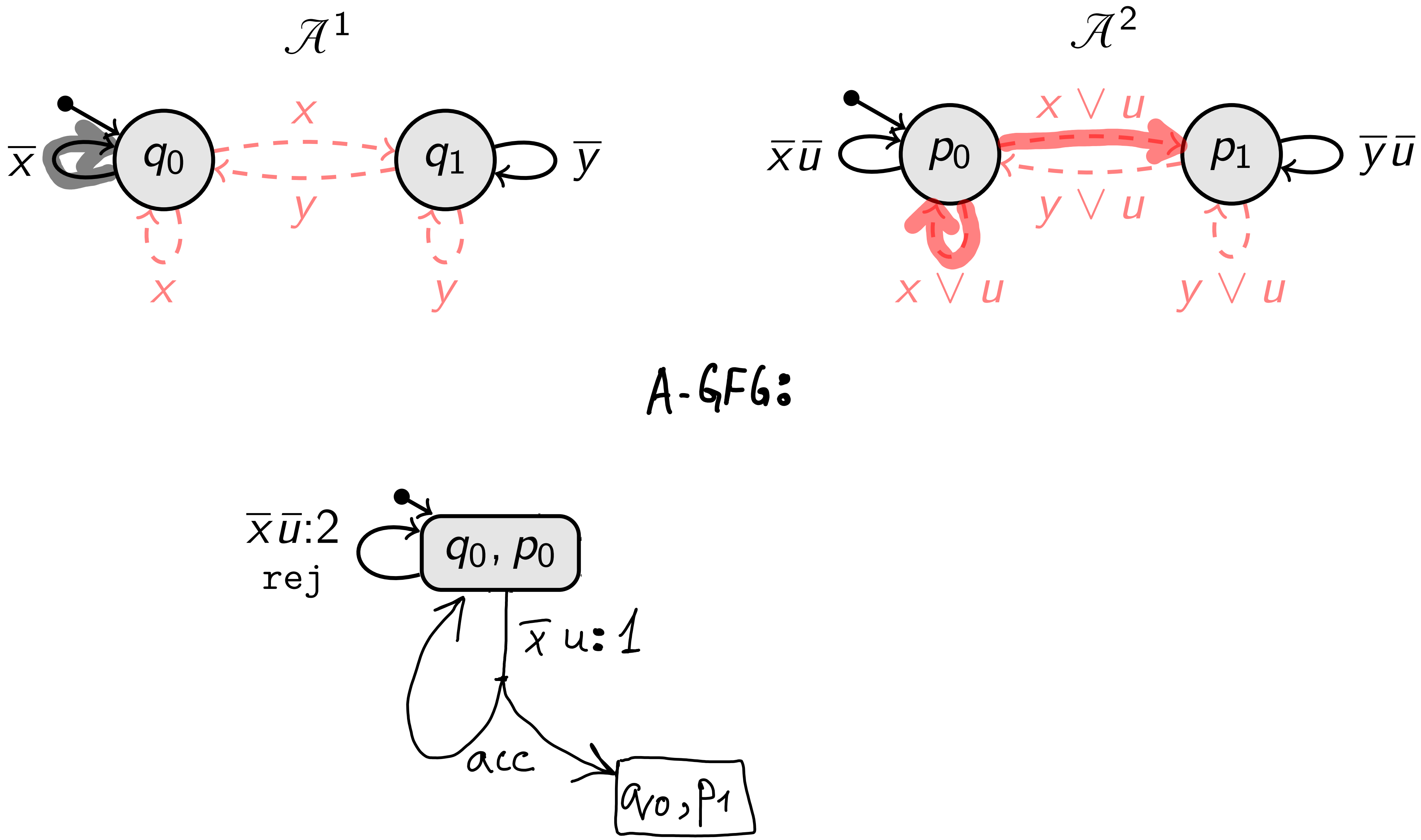

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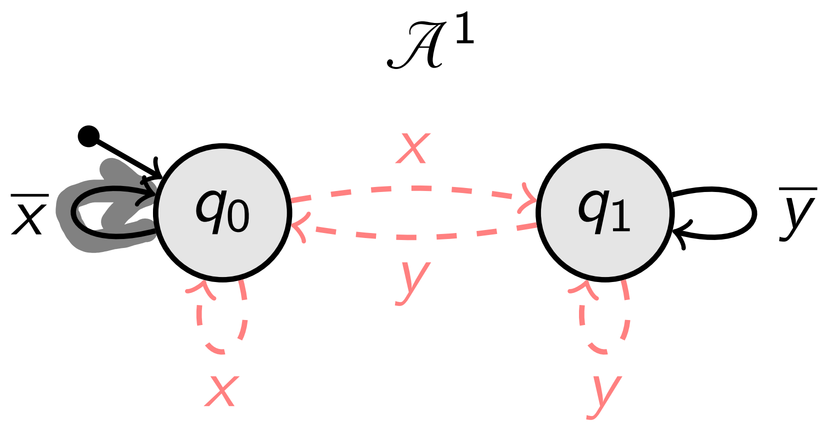

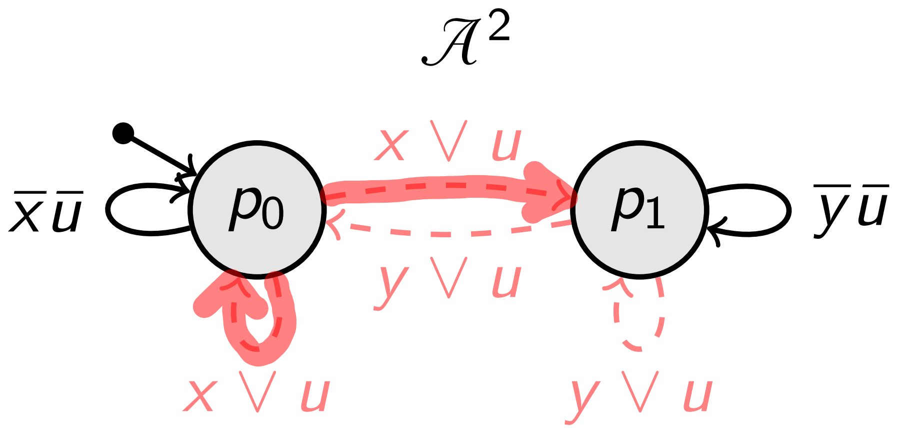

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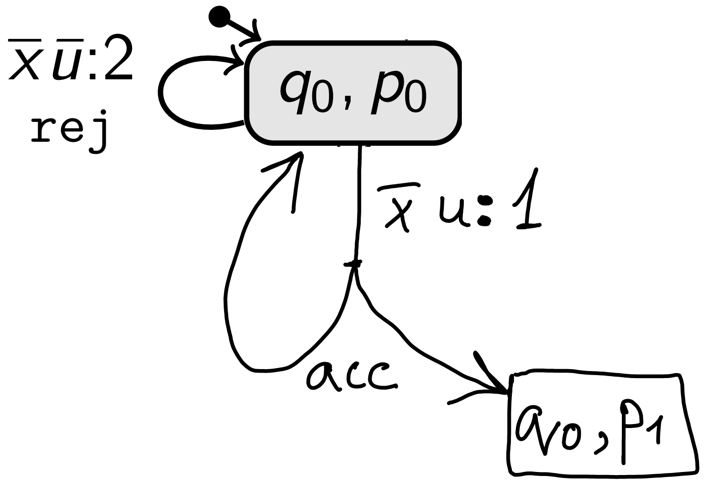

$$L^{good}(q_0 p_0) = (\overline{x}\,\overline{u})^\omega$$

$$L^{good}(q_0 p_1) = (\overline{x}\,\overline{y}\,\overline{u})^\omega$$

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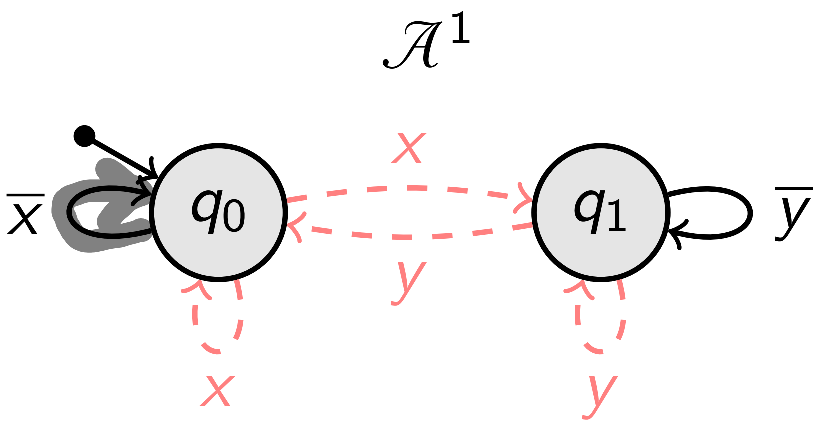

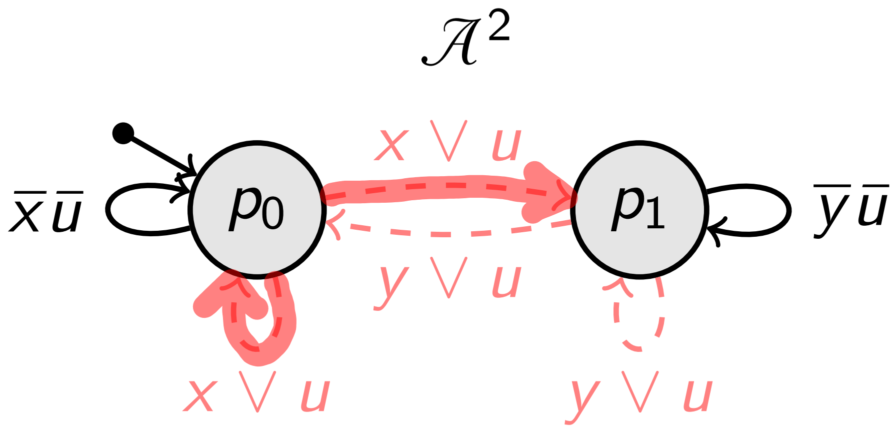

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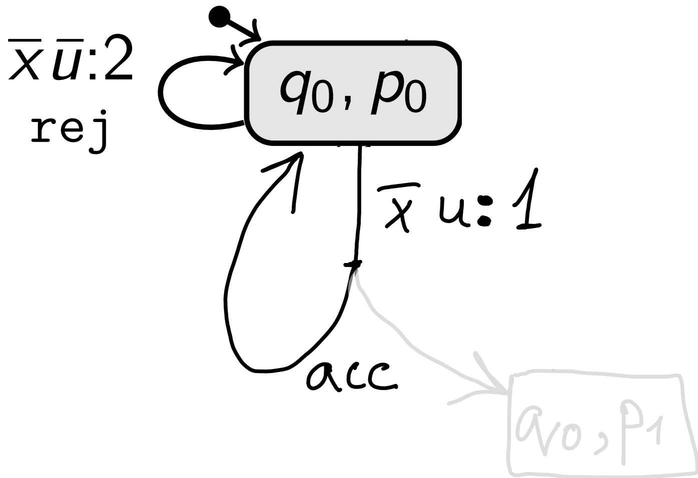

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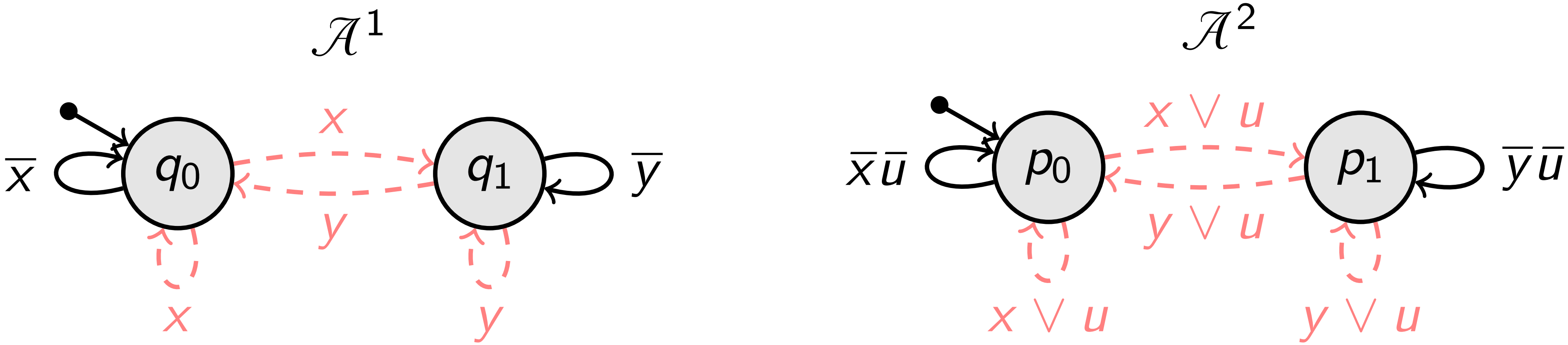

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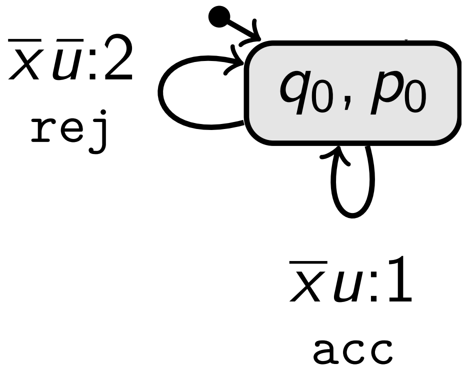

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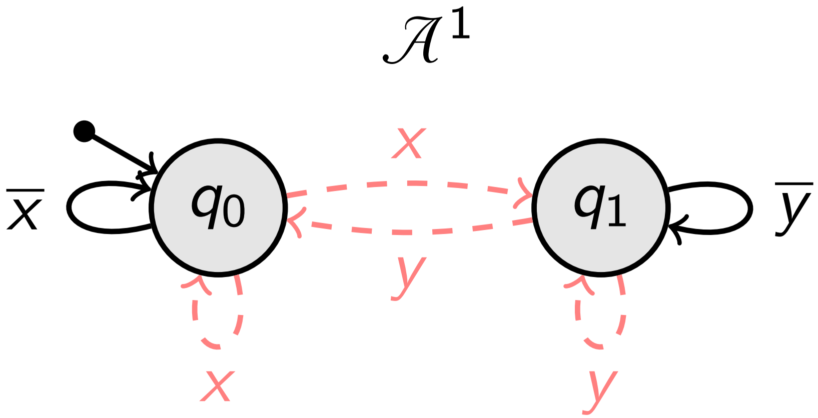

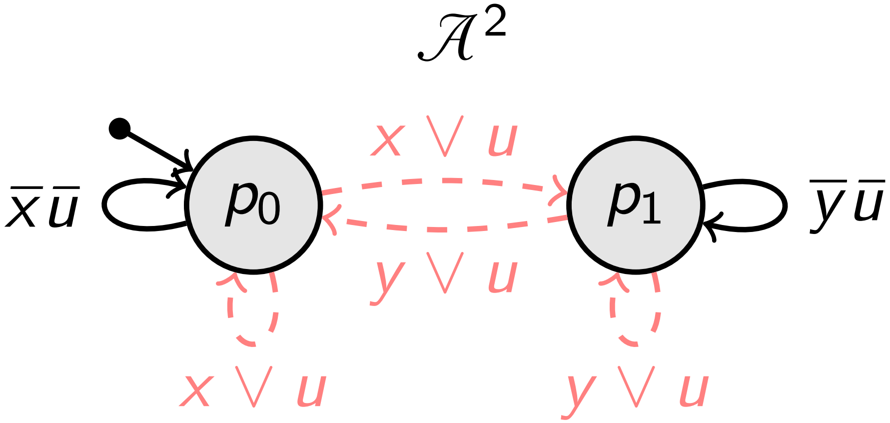

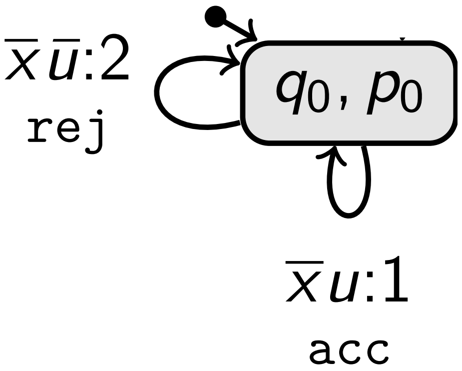

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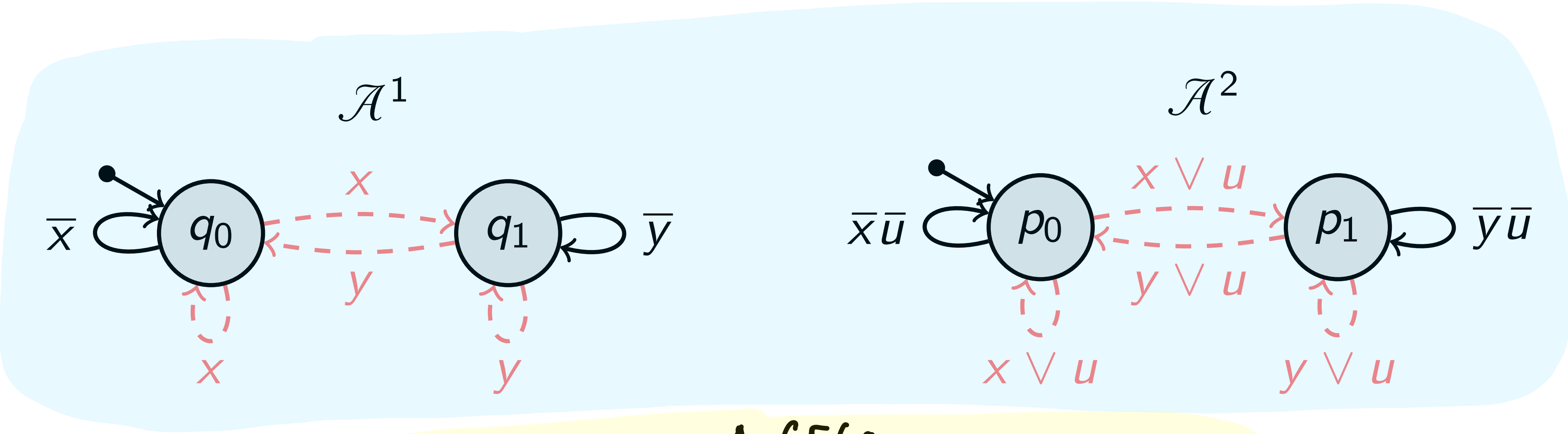

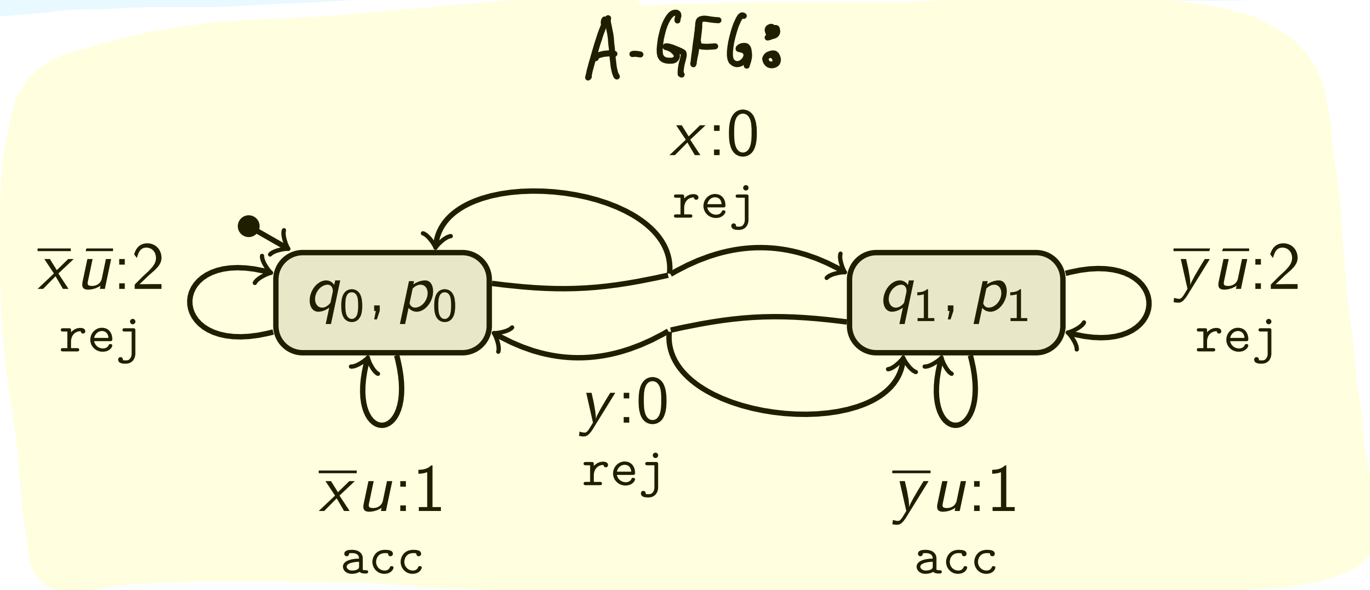

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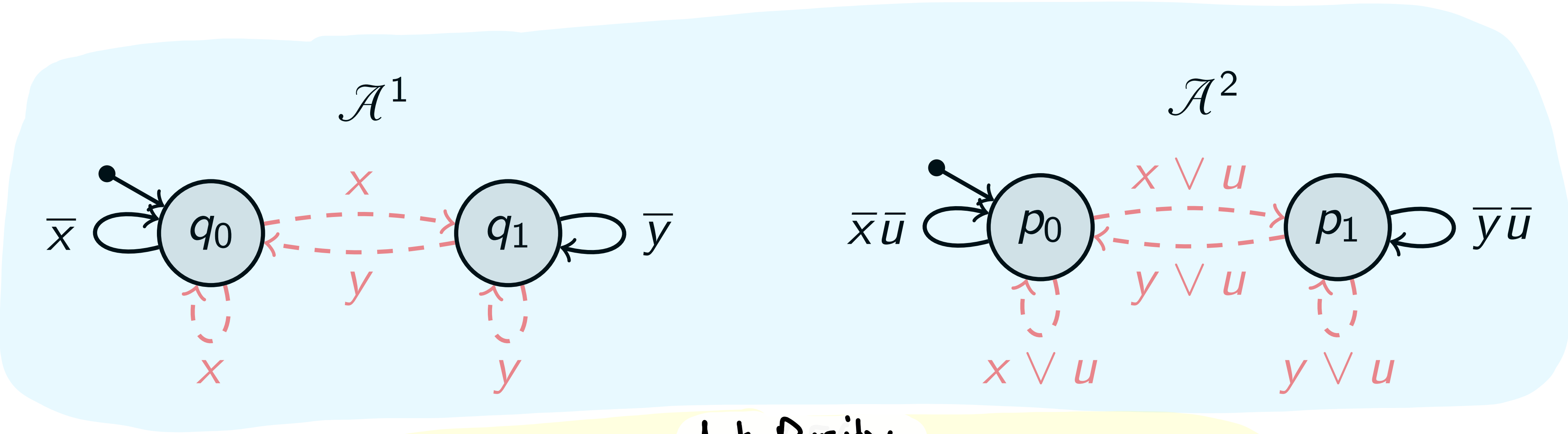

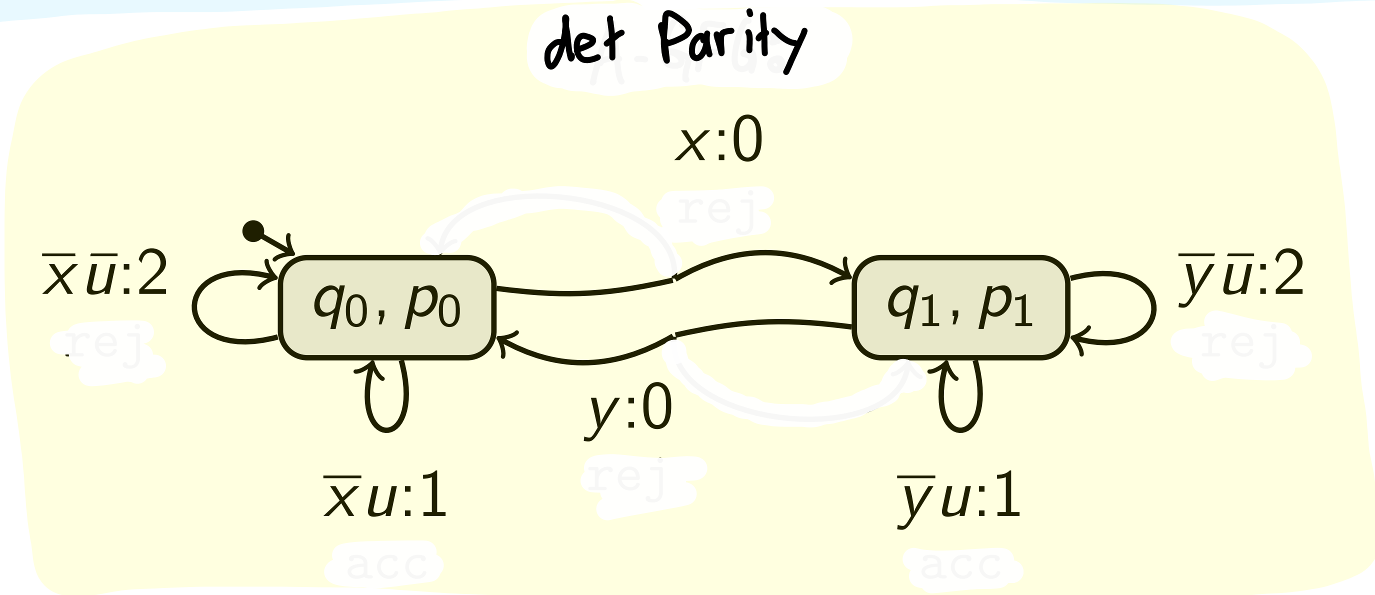

What is the point???

Comparing Fixpoint Formulas for

$$\mathsf{GF}u \to \mathsf{GF}x \wedge \mathsf{GF}y$$

Parity

$$\begin{bmatrix} W_0 \\ W_1 \end{bmatrix} = \nu \begin{bmatrix} Z_0 \\ Z_1 \end{bmatrix} . \mu \begin{bmatrix} Y_0 \\ Y_1 \end{bmatrix} . \nu \begin{bmatrix} X_0 \\ X_1 \end{bmatrix} . \Box\Diamond \begin{bmatrix} xZ_1 & \vee & \bar{x}uY_0 & \vee & \bar{x}\bar{u}X_0 \\ yZ_0 & \vee & \bar{y}uY_1 & \vee & \bar{y}\bar{u}X_1 \end{bmatrix}$$

COCOA

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Acceleration!

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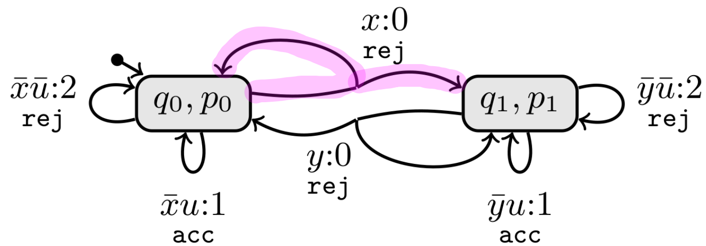

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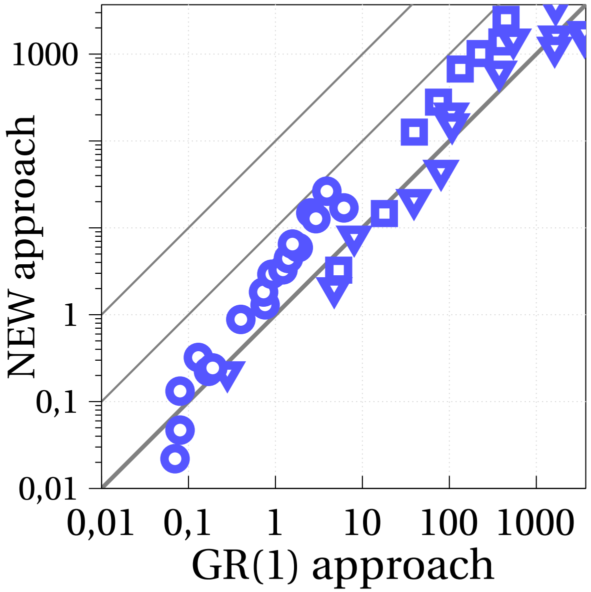
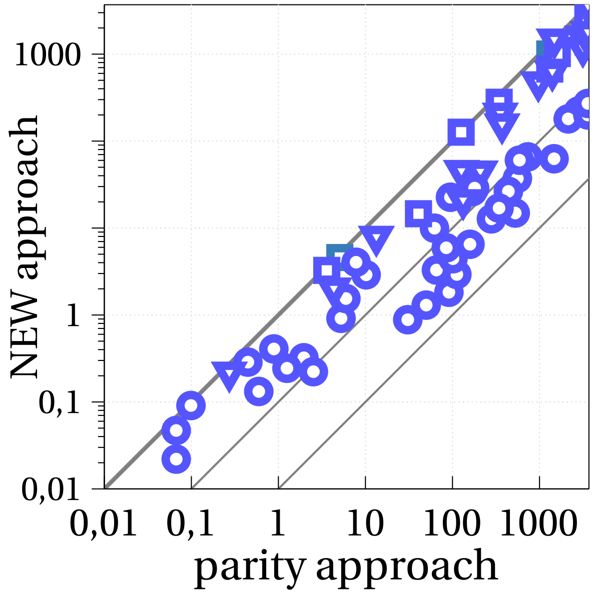
Acceleration!

Evaluation

- parameterized benchmarks: LIFT, AMBA, ROBOT
- prototype tool `Reboot`

Fixpoint evaluation performance:





Conclusion

Problem: Given a symbolic game with LTL objective. Who wins the game?

Φ_{LTL}

↓

parity automaton

↓

COCOA

↓ insight 1

alternating GFG
parity automaton

↓ insight 2

fixpoint formula

↓

evaluate it on the game graph