Bounded Synthesis of Register Transducers

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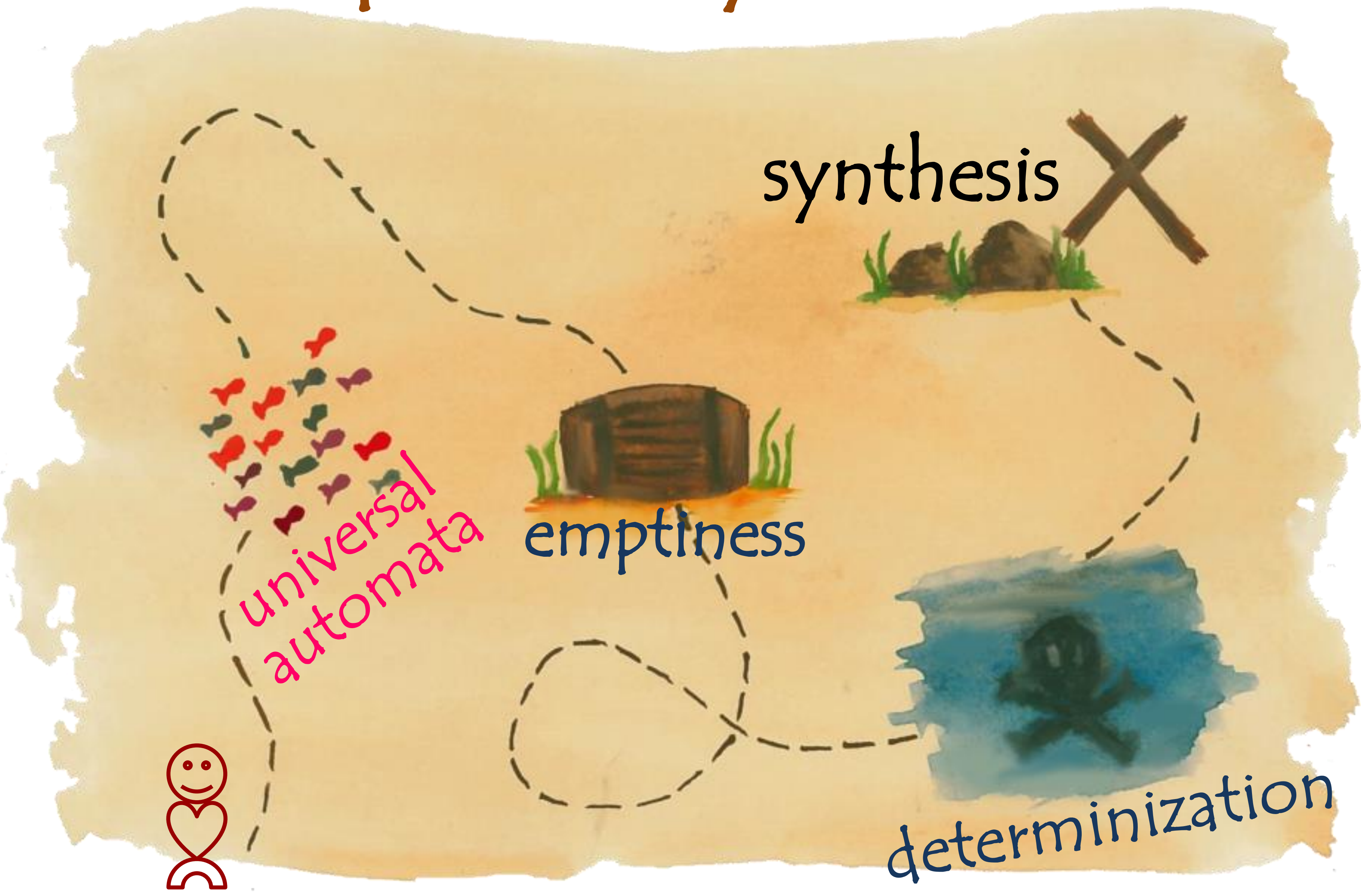
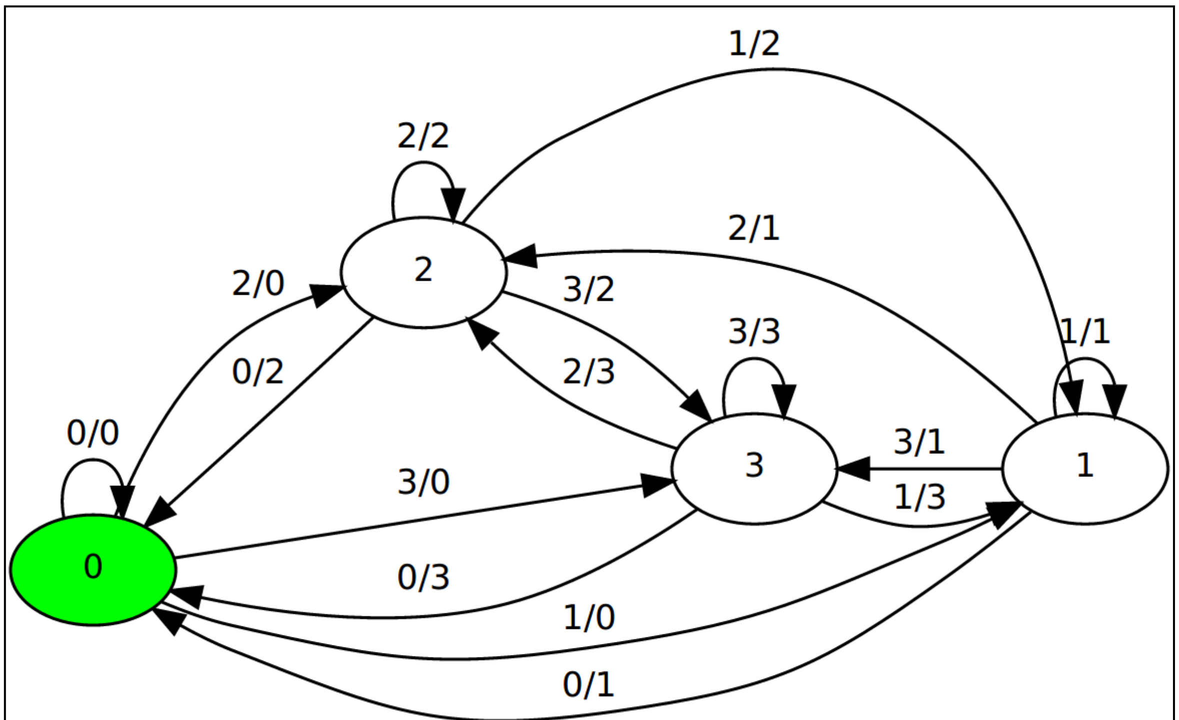
path to synthesis

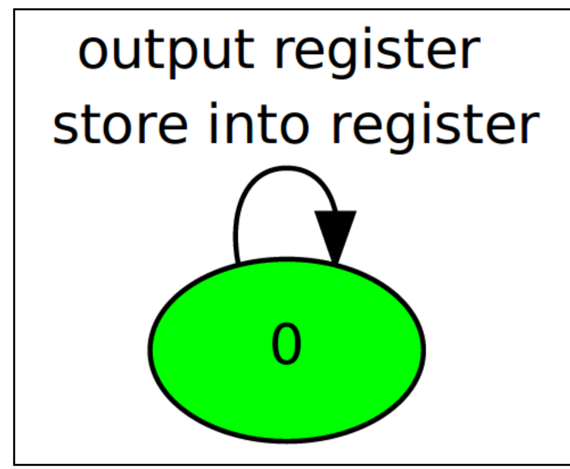


Motivation

Synthesize readable programs.

"Every msg$\in \{0, \dots, 3\}$ should be X-output":





Motivation

Synthesize readable programs.

- Programs separate data/control, but synthesizers do not distinguish. This:
  - makes programs unreadable
  - doesn't scale
  - doesn't work for infinite data domains

=> handle data directly in synthesis

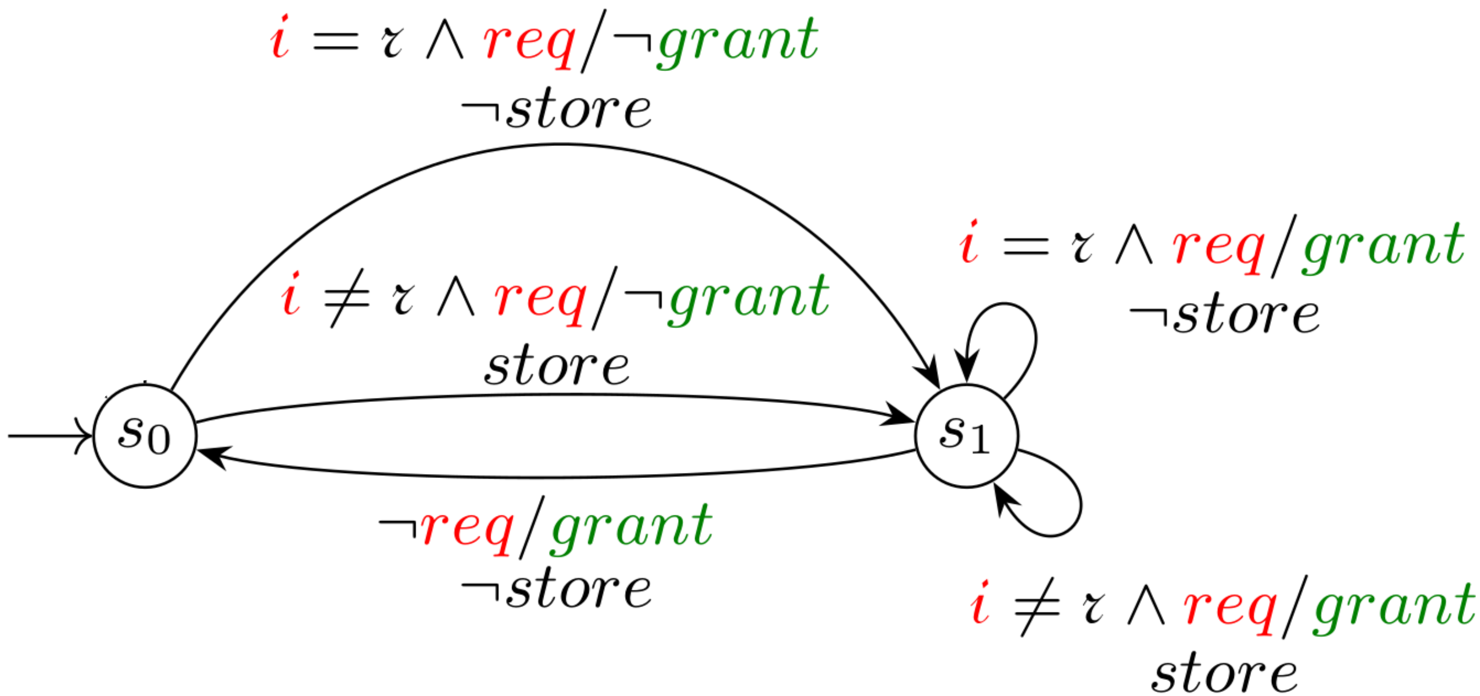
Register Transducers

- Reads a word in $(2^I \times D)^\omega$
- Outputs a word in $(2^O \times D)^\omega$

$$T = (I, O, S, R, s_0, \tau)$$

- Boolean inputs I and outputs O
- States S; s_0 is initial
- Registers $R = \{r_1, \ldots, r_{k_s}\}$, all initialized to d_0
- τ: $S \times 2^I \times B^{k_s} \to S \times 2^O \times \{1, \ldots, k_s\} \times B^{k_s}$

Arbiter (1-Register Transducer)



- Input: $(req, 1)(\neg req, 2)(\neg req, 3)$ …
- Run: $(s_0, 0) \xRightarrow{\neg g, 0} (s_1, 1) \xRightarrow{g, 1} (s_0, 1) \xRightarrow{\neg g, 1} (s_0, 1)$ …

Register Automata (Universal CoBuchi)

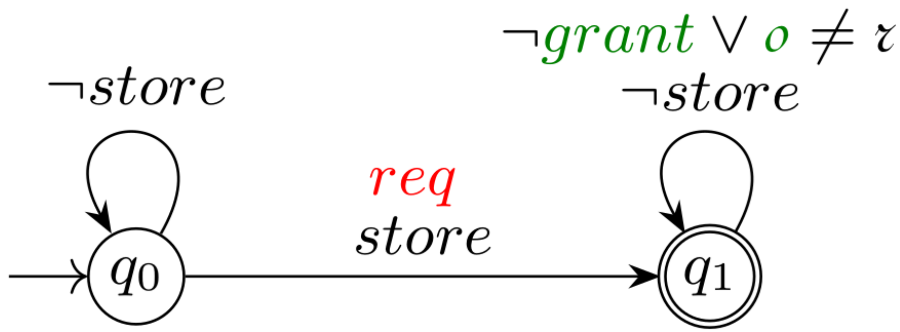
- Works on words in $(2^P \times D \times D)^\omega$

$$A = (P, Q, R, q_0, \delta)$$

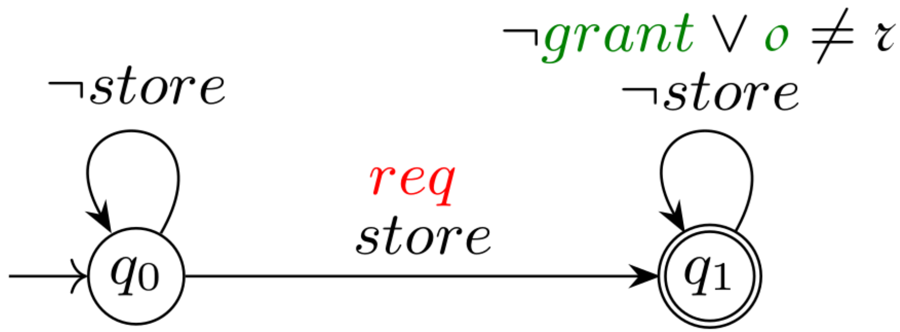
- P are Boolean signals
- States Q; q_0 is initial
- Registers $R = \{r_1, \dots, r_{k_A}\}$, initialized to d_0
- δ: $Q \times 2^P \times B^{k_A} \times B^{k_A} \to 2^{Q \times B^{k_A}}$

Spec for Arbiter (1-Register Automaton)

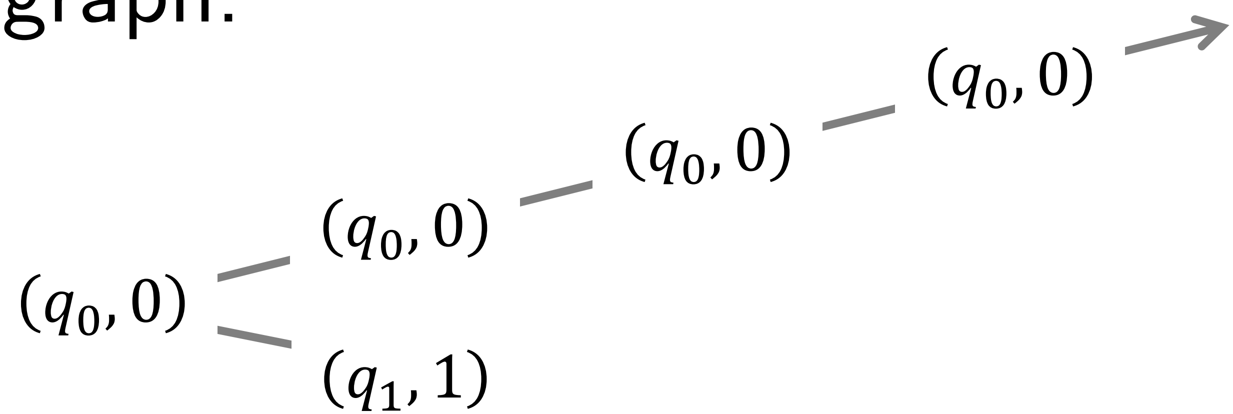
$$\forall d \in D\colon\ \mathbf{G}(req \wedge i = d \ \rightarrow\ \mathbf{X}\,\mathbf{F}(grant \wedge o = d))$$



1-Register Automaton



- Word: $\binom{req, 1}{\neg grant, 0} \binom{\neg req, 2}{grant, 1} \binom{\neg req, 3}{\neg grant, 1} \ldots$
- Run-graph:



Bounded Synthesis Problem

Given:

- I, O
- register automaton A over I and O
- the number k_s of registers in a system

Return:

- k_s-register transducer sys such that $sys \models A$, or "unrealizable"

Bounded synthesis problem is decidable in $2^{const\cdot|Q|\cdot 2^{(k_A+k_s)^2}}$ time.

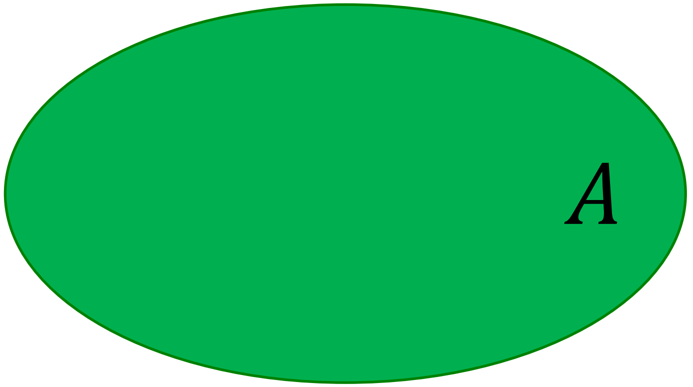
Synthesis

Standard approach:

$$A$$

$$\Downarrow$$

$$\det A$$



Synthesis

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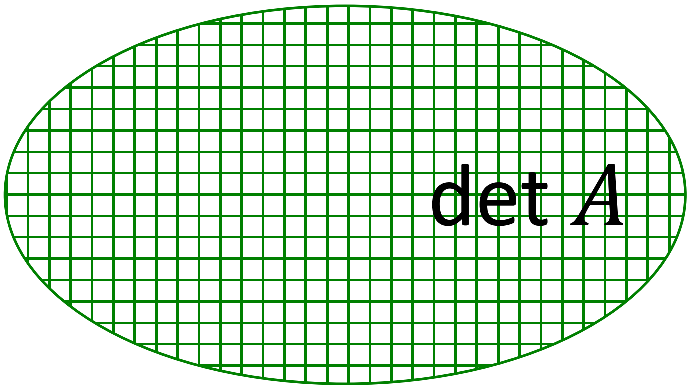


Synthesis

Standard approach:

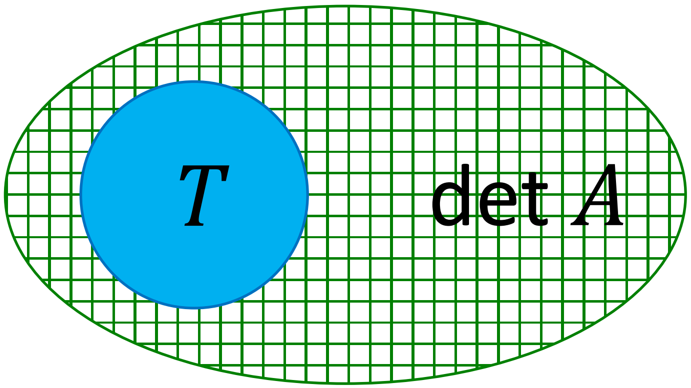
$$A$$

$$\Downarrow$$

$$\det A$$

$$\Downarrow$$

find T



Synthesis

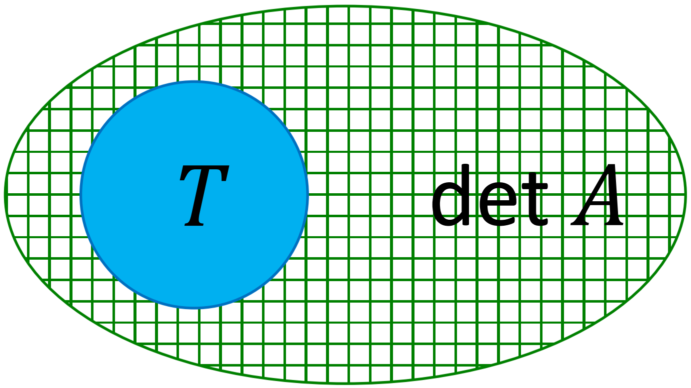
Standard approach:

A

undecidable for register automata

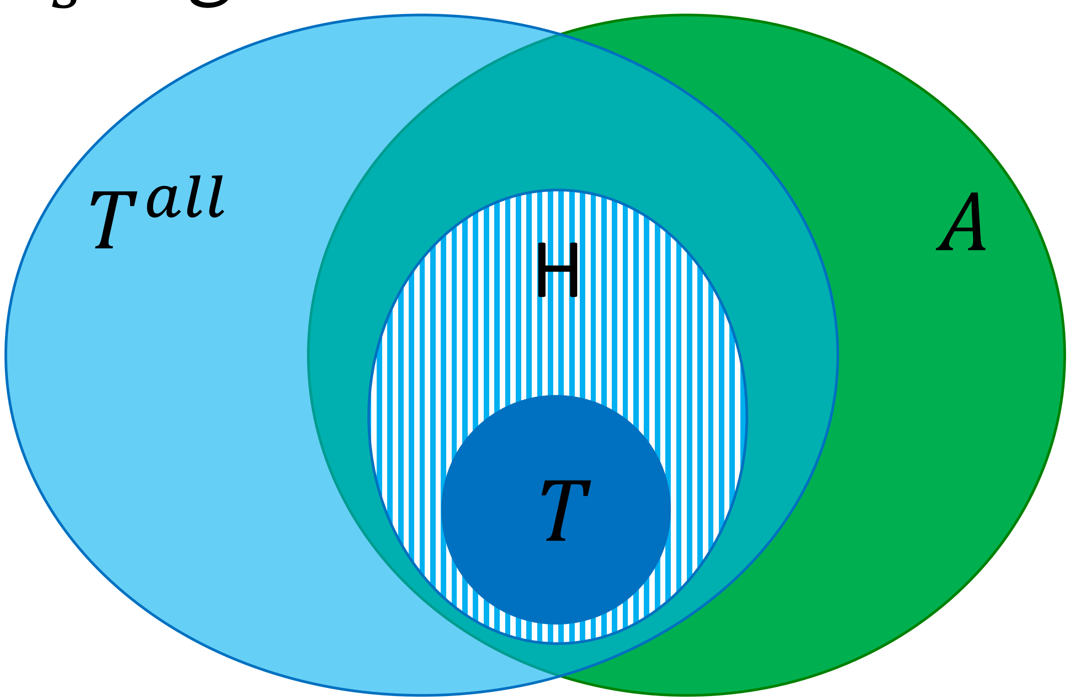
det A

find T



Synthesis

Our approach focuses on words generated by k_s-register transducers $A \cdot T^{all}$.

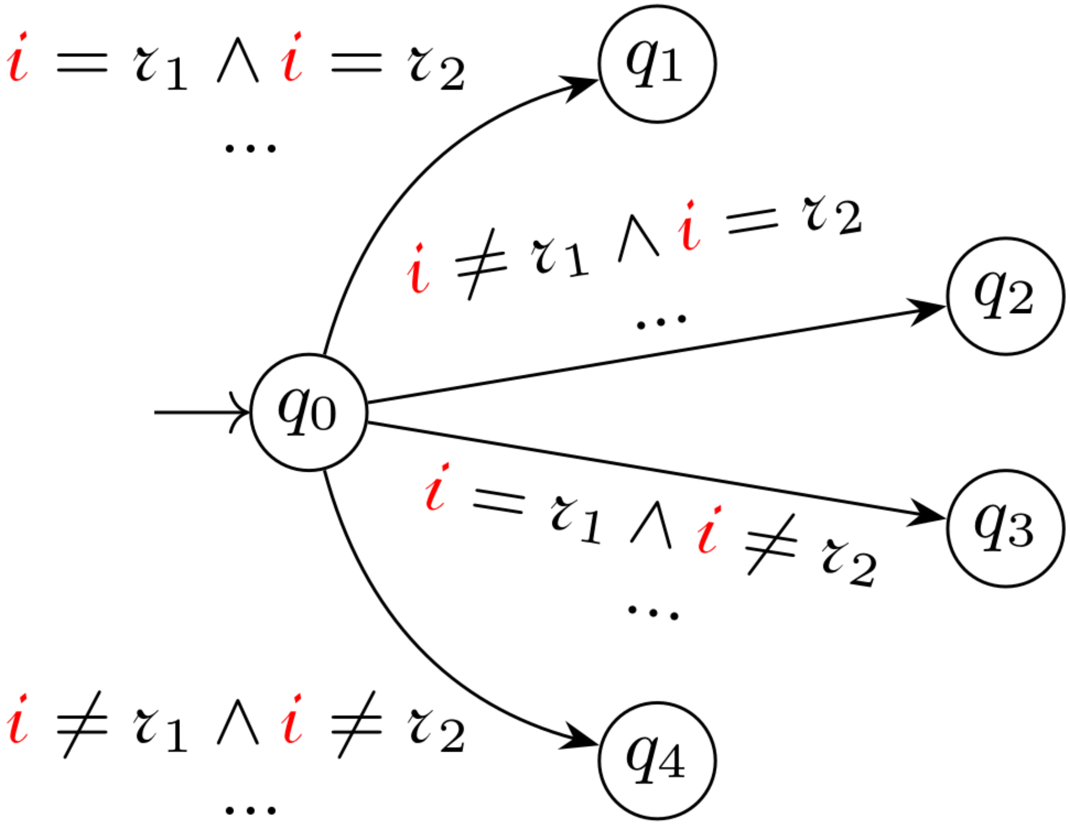


Abstraction for Model Checking

Check rej-emptiness of $sys \times A$

Abstraction for Model Checking

Find a rejecting word in a register automaton



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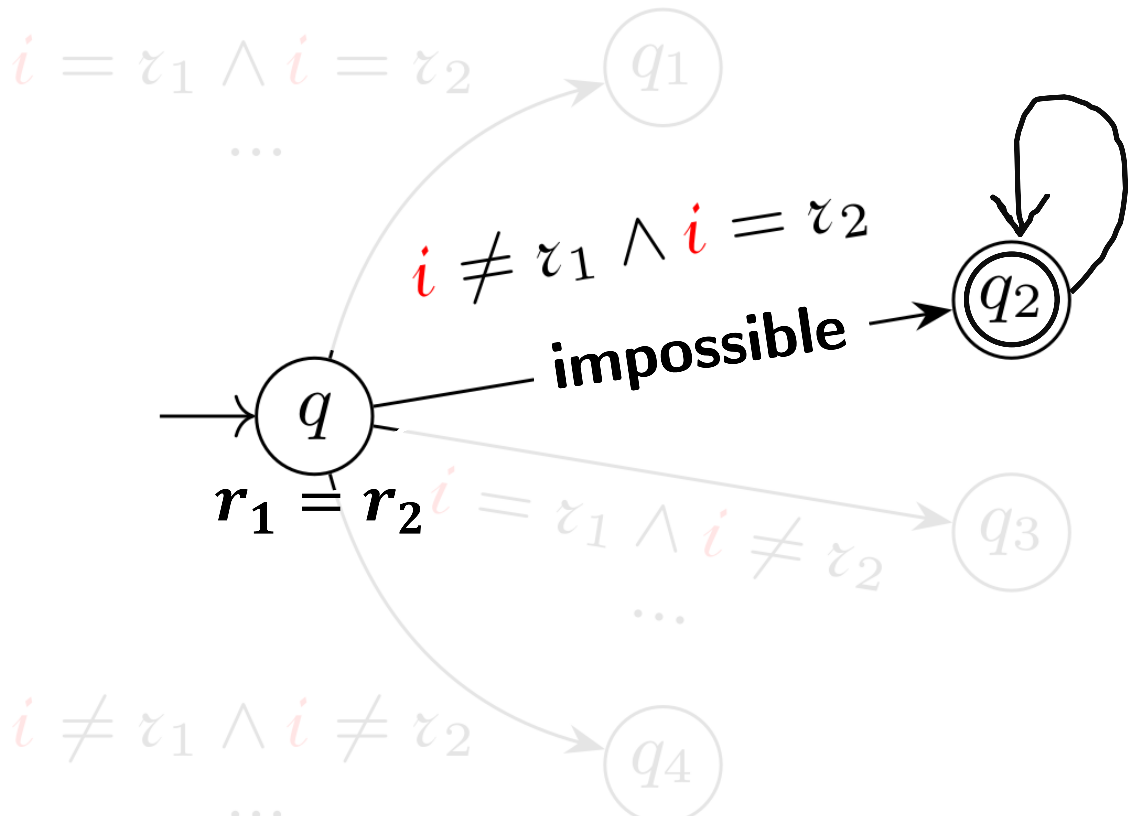

Abstraction for Model Checking

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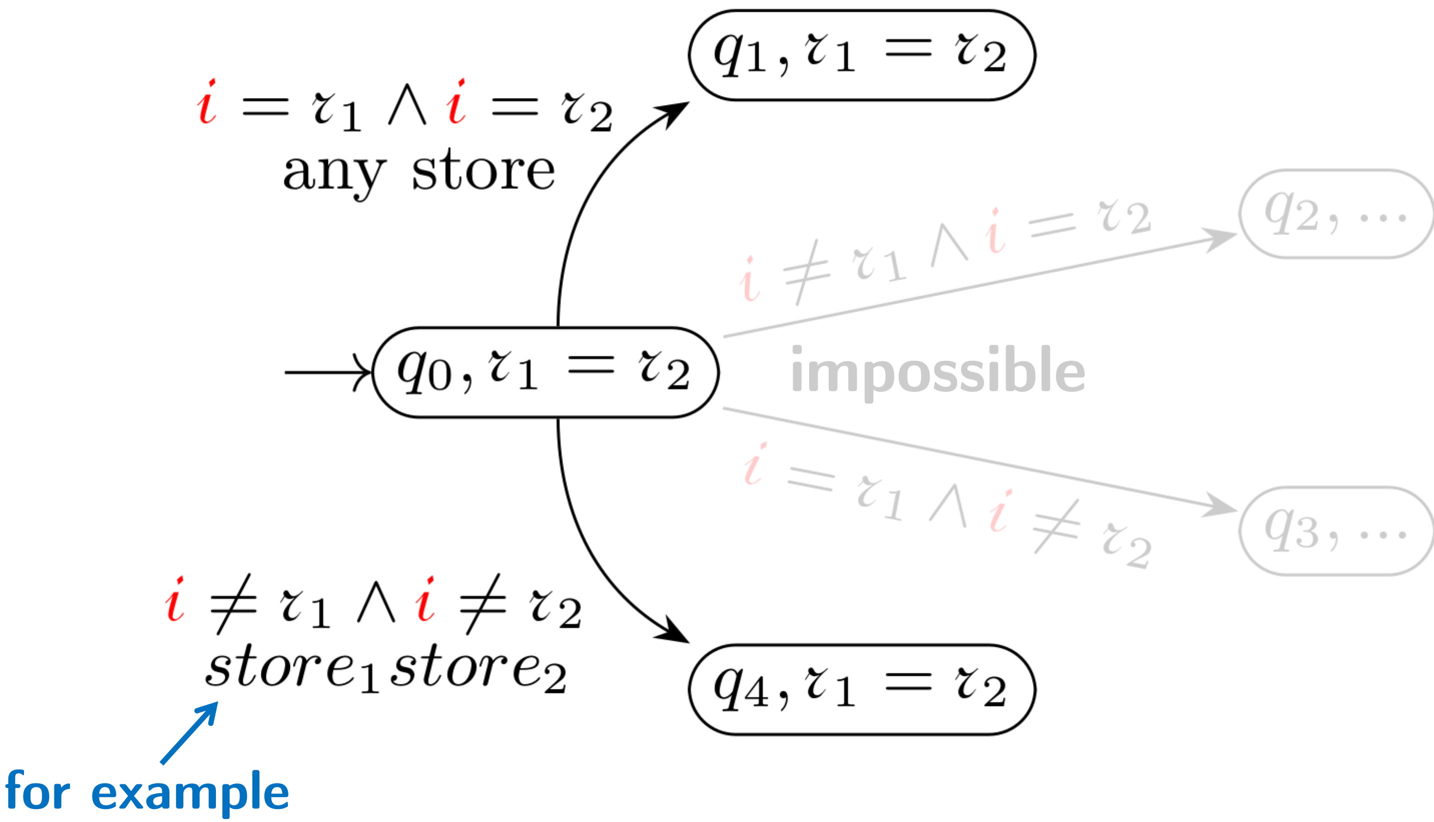
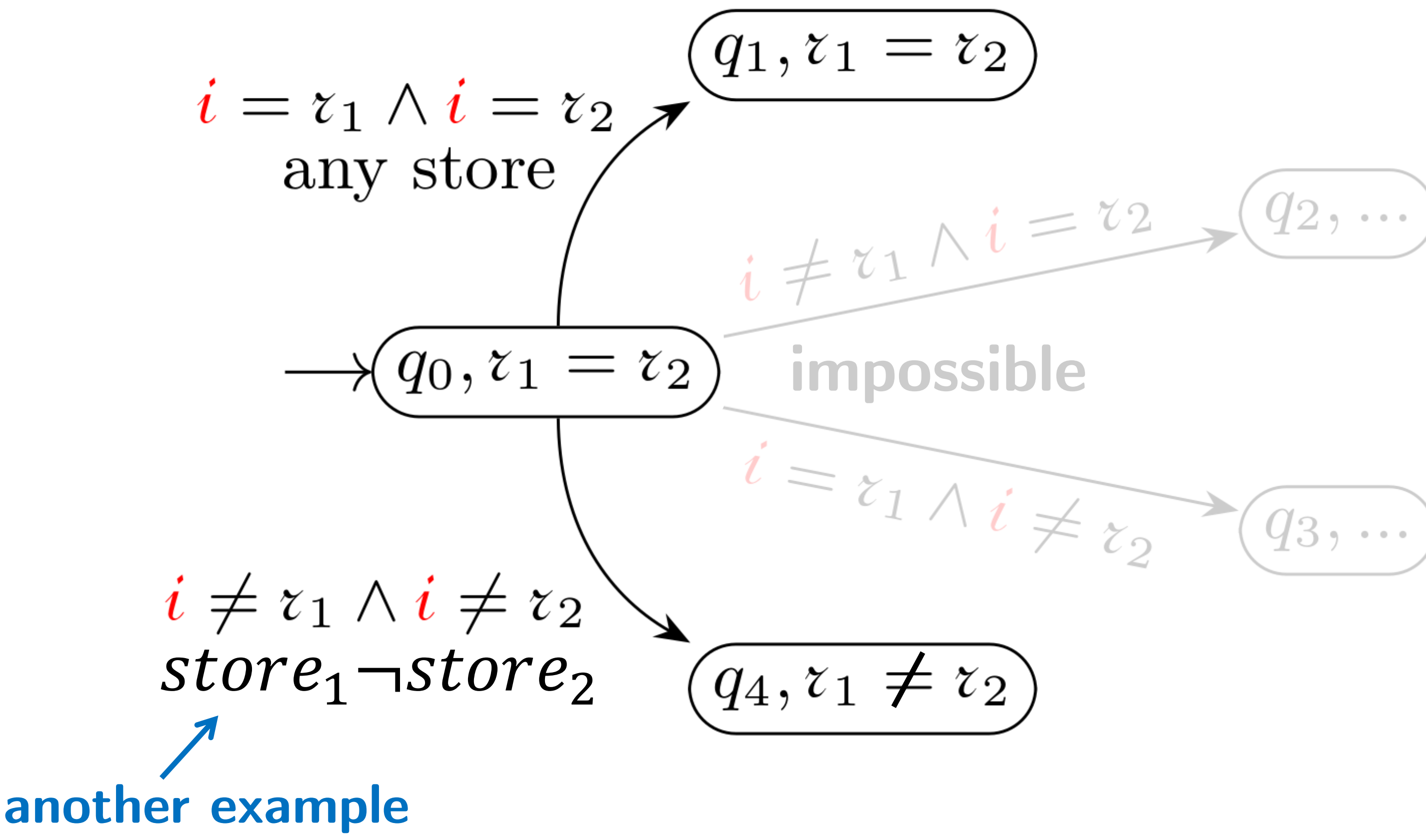


Abstraction for Model Checking

Find a rejecting word in a register automaton



Abstraction for Model Checking

Find a rejecting word in a register automaton

==

*find a rejecting word in register-**less** abstraction*

$$(sys \times A)@V$$

$$
\begin{aligned}
(r_1 = r_2)' \quad \Leftrightarrow \quad & store_1 \land store_2 \lor \\
& i = r_1 \land store_2 \lor \\
& i = r_2 \land store_1 \lor \\
& (r_1 = r_2) \land \neg store_1 \land \neg store_2
\end{aligned}
$$

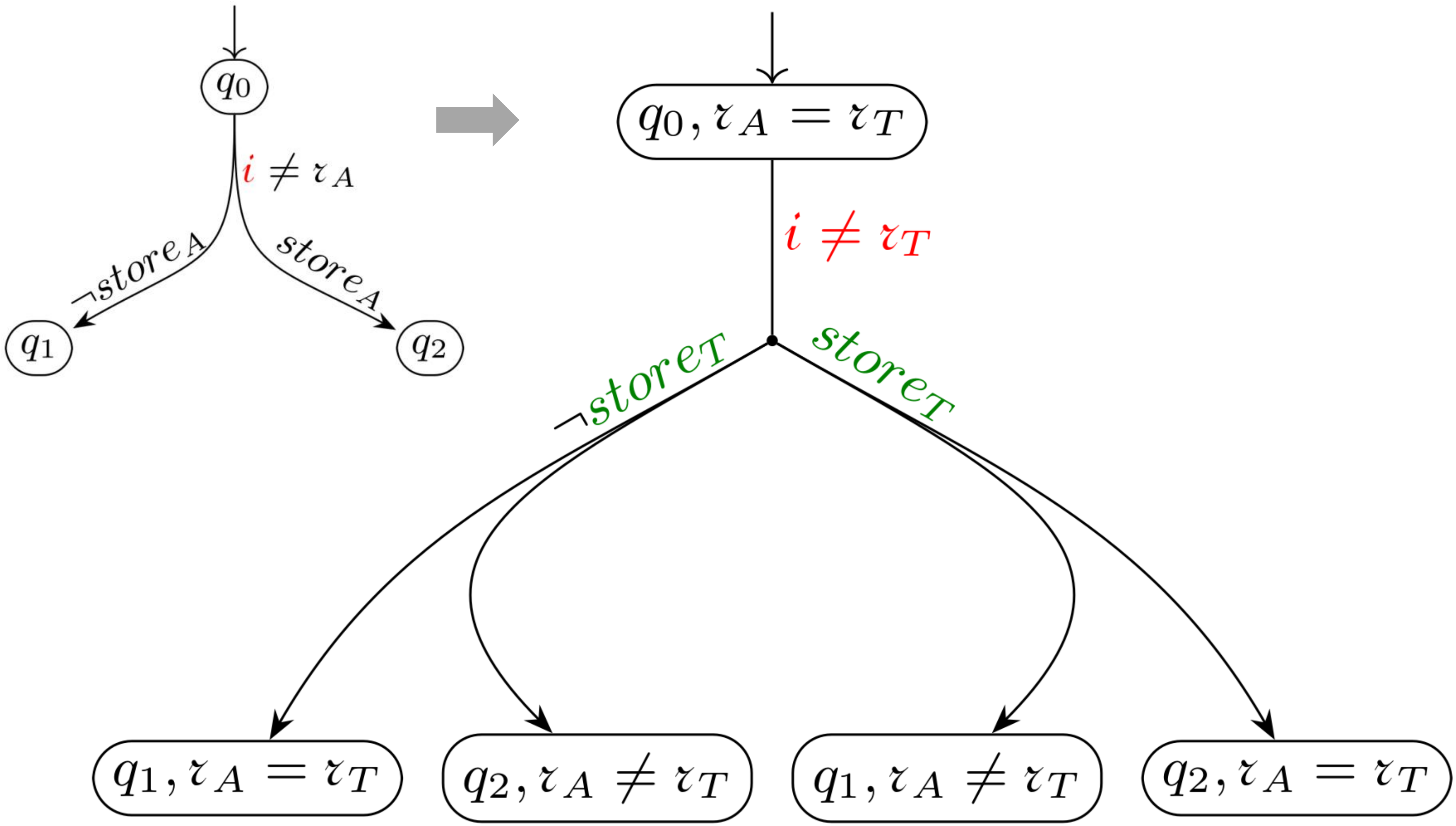
Synthesis

- System is not given $\cancel{(sys \times A)@V}$
- We cannot rely on the registers of A, bcz their assignments are universal $\cancel{A@V}$**?**

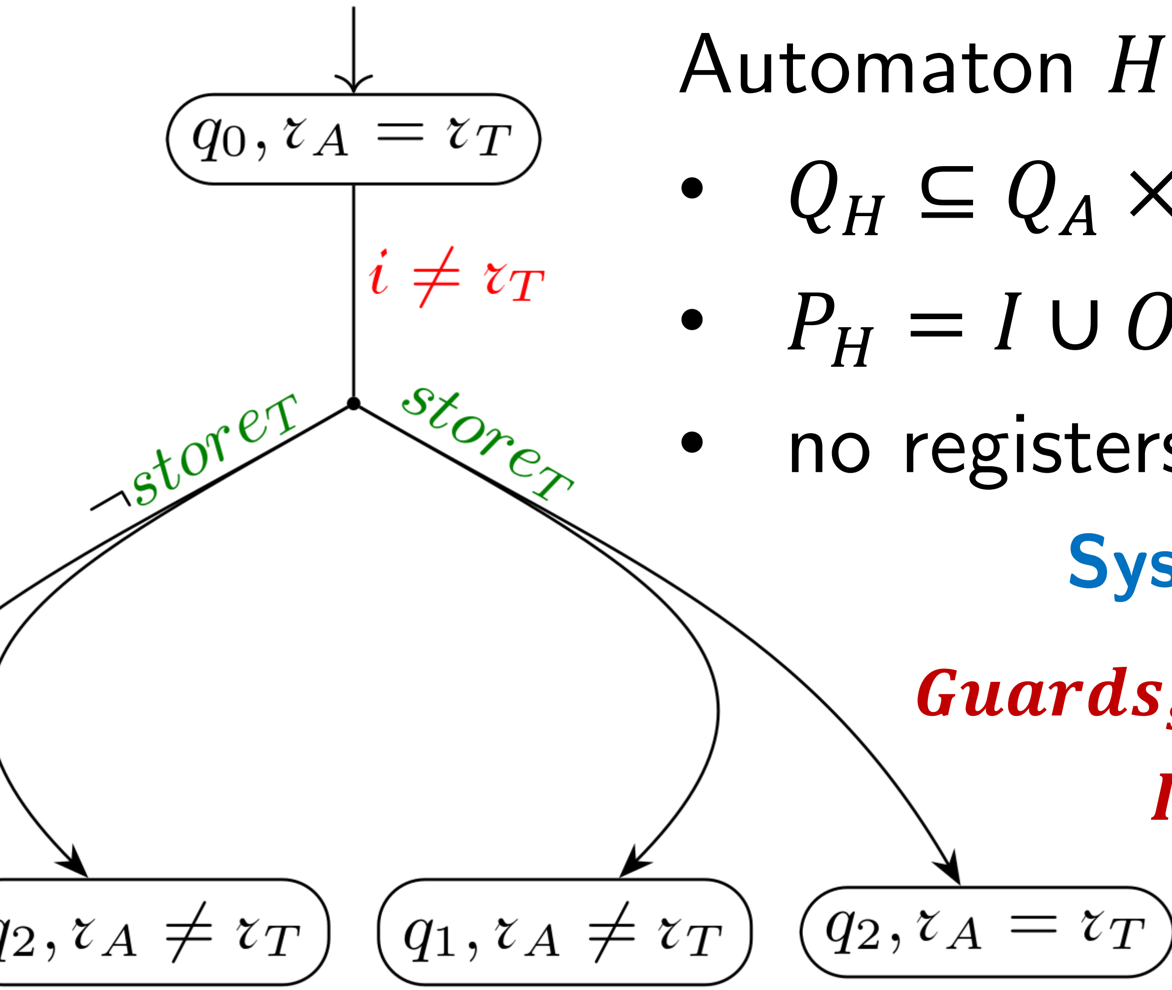
Introduce new registers controlled by transducers.

Synthesis

Introduce new registers controlled by transducers.



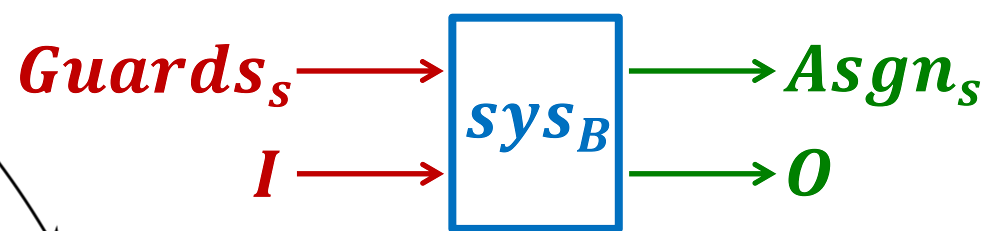
Abstraction for Synthesis



Automaton H:

- $Q_H \subseteq Q_A \times \Pi_{R_A \cup R_s}$
- $P_H = I \cup O \cup Guards_s \cup Asgn_s$
- no registers

Systems are Boolean:

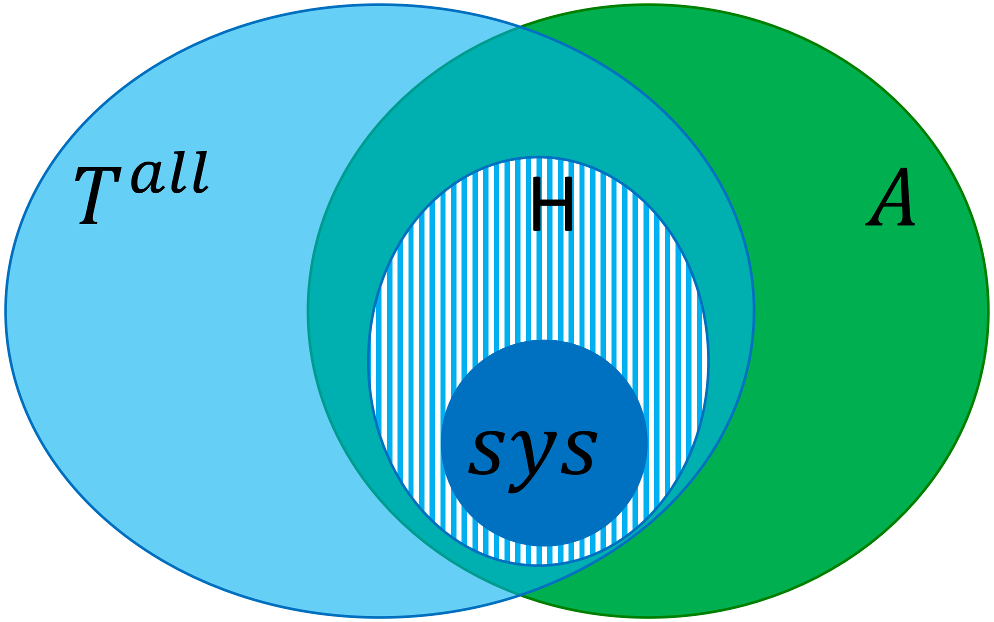


$$sys \vDash A \iff sys_B \vDash H$$

Synthesis

For every A, k_s:
$\exists sys$: $sys \vDash A \iff \exists sys_B$: $sys_B \vDash H$

- Time complexity: in $2^{const \cdot |Q| \cdot 2^{(k_A+k_S)^2}}$



Conclusion

- Contributions
  - Decision procedure for bounded synthesis of register transducers from universal register automata
  - (Not presented) Incomplete procedure for bounded synthesis of register transducers from $\forall$LTL(EQ)
- Possible future directions
  - lower complexity bound
  - non bounded synthesis
  - richer automata and systems